DETERMINING BACK-FACING CURVED MODEL SURFACES BY ANALYSIS AT THE BOUNDARY

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Determining Back-facing Curved Model Surfaces By Analysis At The Boundary

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Abstract

This paper examines the problem of predicting when a model surface patch on a three-dimensional object is totally back-facing, and hence need not be searched for during object recognition. Examining every point on the surface patch is inelegant and impractical, yet difficulties arise with curved surface patches. The paper concludes that visibility can be determined from an analysis of the surface orientation at the patch boundary for a wide class of model surface patch types (i.e. those having constant principal curvature signs), under orthographic projection.

1 Introduction

A typical problem for three dimensional scene analysis [2] is determining the visibility status of a particular feature. This usually occurs during a model verification phase, when a reference frame has been estimated for a model, and it is then desired to verify the model by finding additional image evidence. One approach to finding additional evidence is to use the estimated reference frame with a geometric model to predict the location of previously unmatched features, in this case, surfaces.

When working in 3D scenes, this problem is complicated because some surfaces will lie on the back sides of objects where they are not visible (surfaces may also be self-occluded, but that issue is not considered here). Hence, it is important to determine whether the surface is even visible, before searching for evidence for it. (We assume that surface patches are one-sided, with an outward-facing surface normal.)

The visibility of a patch can be classified (in the absence of occlusion - self or external) as:

classification	visibility
back-facing	completely invisible surface
front-facing	completely visible surface
tangential	partially visible surface
degenerate tangential	seen as a curve

For this paper, we are only concerned with identifying the back-facing surface patches.

If all surfaces are planar, then determination of visibility is easy: if the outward normal of a planar surface faces away from the viewer, then the surface is back-facing. On generic curved surfaces, it is not possible to determine visibility from analysis at only a single point on the surface, because it is possible for the surface to curve away from a back-facing point to become forward-facing elsewhere. Moreover, even if we know the visibility all along the patch boundary, or at any incomplete set of points, the surface can still deform internally to peek out from behind. Hence, the general case cannot be solved.

Fortunately, if we make some restrictions on the shape of the surface patch primitives, then it is possible to make some conclusions. We show below that **if the patch is a surface with constant principal curvature signs, and if every point on the boundary of the patch is back-facing, and the patch is not a *bubble patch* (defined below), then the whole patch must be back-facing.**

In the analysis below, we treat the surfaces as if made of "tinted air" (in the words of Koenderink [4]). Hence, the **full extremal boundary** is:

Definition 1 (Full Extremal Boundary) *The set of all points on the surface with the surface normal tangent to the line of sight (i.e. even when occluded by closer surface points) under orthographic projection.*

As Koenderink [4, pg 420] says: "For a smooth object, it is a smooth space curve; moreover, it is *closed*, although it may be composed of several disconnected loops."

We also use the term "full contour":

Definition 2 (Full Contour) *The orthographic projection of the full extremal boundary onto the image plane.*

Then, the *bubble patch* is:

Definition 3 (Bubble Patch) *A surface patch is a 'bubble patch' if: (1) it is a portion of a surface with positive gaussian curvature and (2) it is possible, for a viewer under orthographic projection, to observe the patch such that a closed non-degenerate full extremal boundary can be placed on the patch, enclosing a forward-facing subset of the patch.*

A patch that consists of the inside (or outside) of a sphere except for a hole (where the back-facing normals are found) is a *bubble patch.* Figure 1 illustrates an example of this patch.

Each of the patches considered below could have internal holes in them, by which we mean that the surface patch may have more than one boundary. However, since we show that the external boundary determines if the patch is back-facing, then all interior points must be back-facing. Hence, the boundaries of new interior holes can only be back-facing, too.

There may also be orientations of the patches where forward-facing portions of the patch are not visible, because they are obscured by other portions of the patch. This case is not considered, because here the invisibility is caused by *occlusion* rather than purely patch orientation.

2 The Visibility of Constant Curvature Sign Patches

Constant curvature sign patches are patches whose principal curvature signs are constant over the patch. We need only consider only the three signs ('+', '0', '-') for each of the two principal curvatures, so this defines six surface shape classes: '++', '+0' = '0+', '+-' = '-+', '00', '--', '-0' = '0-'. We now show that surface patches taken from these shape primitives satisfy Theorem 1.

The proof uses a result from [3], that showed that the product of the curvature of the full contour and the radial curvature of the surface along the line-of-sight is equal to the Gaussian curvature. We use this to determine the local shape class of the surface at the boundary. That is, the product of the signs of the full contour's curvature and the sign of the radial curvature must equal the sign of the Gaussian curvature, which we know from the shape of the model surface whose visibility we are analyzing.

Definition 4 (Constant Curvature Sign Patch) *A patch is a "constant curvature sign surface patch" if it is C^2 and the signs of its two principal curvatures do not change over the patch.*

Theorem 1 *If (1) the surface normal at every point on the boundary of a finite constant curvature sign surface patch is back-facing, (2) the patch is viewed from a single viewpoint projected orthographically onto an image plane (i.e. is not simultaneously viewed from more that 180 degrees) and (3) the patch is not a 'bubble patch', then the whole patch must be back-facing.*

Proof:

Assume that we have a constant curvature sign C^2 surface patch oriented with backward-facing normals around the patch boundary. We will show that if there is a forward-facing patch internal to the patch boundary, then the patch must be a *bubble patch.*

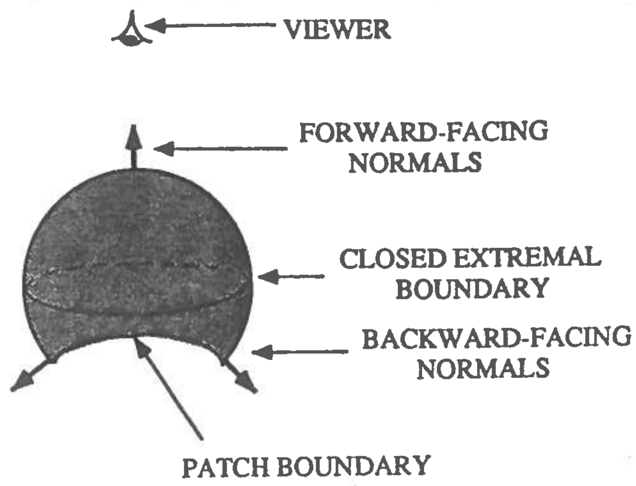


Figure 1: Bubble Patch Example

If there is an internal forward-facing sub-patch, then it must be enclosed by a full extremal boundary within the whole patch (as the patch must proceed smoothly from forward-facing internally to backward-facing at the patch boundary on a C^2 surface). Refer to Figure 2.

Then, because the full extremal boundary is closed, it must project onto the image plane either as (a) a closed full contour (note, the *full extremal boundary* is a 3D space curve, whereas the *full contour* is the corresponding 2D image curve) or (b) a degenerate curve enclosing no area. In the second case, the patch internal to the full extremal boundary must then be everywhere tangential to the line of sight, and thus cannot be forward-facing.

Because the surface is C^2 smooth, the full contour must be smooth almost everywhere (except at a finite number of cusps where the projection direction lies in the surface). (The curve may cross over itself, but this is unimportant, as we will only consider the boundary and the surface patch locally adjacent to it.)

Consider now where the forward-facing surface must exist relative to the full contour. If it is on the outside (as when looking through the inside of a torus), then there must be another full extremal boundary outside of it because the forward-facing portion must eventually return to back-facing at the patch boundary. The proof will only need to consider the outer full extremal boundary and so we ignore the inner full extremal boundary. Hence, we need only consider the case when the patch projects to inside the closed full contour.

Then, the full contour must consist of convex, concave and straight segments (segmented at inflexions and cusps), where the convexity of the segment is determined by the side of the curve that the forward-facing surface patch lies on. Refer to Figure 3, where the shading indicates where the interior of the projected patch lies.

By Koenderink's result [3] (and noting that his results also apply to concave hollows, by negating the curvatures obtained assuming the surface were really a convex bump),

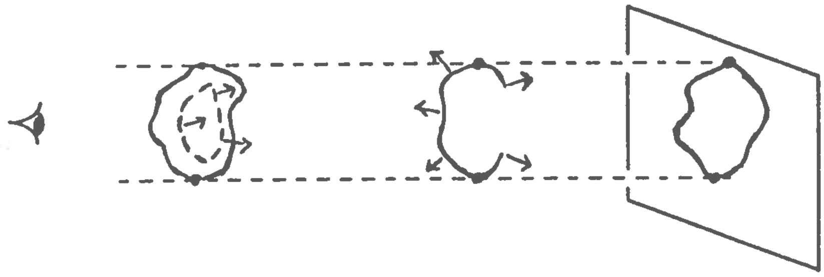


Figure 2: Back-facing Patch With An Internal Forward-facing Sub-Patch

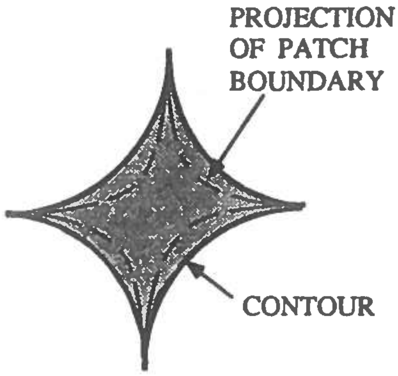
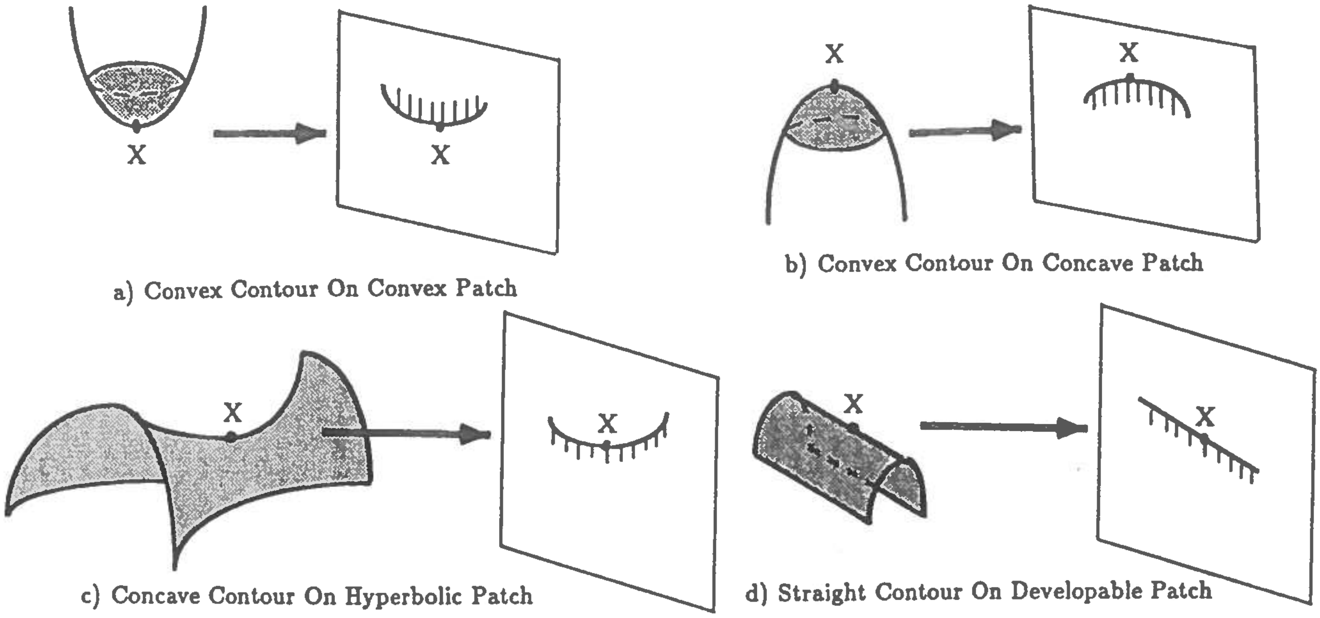


Figure 5: Projection Of Patch With Shrunken Boundary



a) Convex Contour On Convex Patch

b) Convex Contour On Concave Patch

c) Concave Contour On Hyperbolic Patch

d) Straight Contour On Developable Patch

Figure 3: Extremal Boundary Projects To Contour

we can deduce the local shape of the full contour and in which direction the surface bends at the full extremal boundary, from the sign of the Gaussian curvature (which we know from the type of model patch). Hence, a concave segment (concave relative to the region inside the full contour) implies that the local surface curvature along the segment must be '+-'. Similarly, a convex segment implies that the local surface curvature must be either '++' or '--', depending on whether the surface faces inward or outward (Figure 3 parts a and b). A straight segment implies the surface must be developable, with curvature '0+' or '0-' (the case '00' does not occur because it cannot have a full extremal boundary). Because the surface has constant curvature sign, this implies that the projected curve must consist of either all convex, all concave or all straight segments. We consider each case separately below.

Consider an arbitrary set of convex full contour segments (e.g. Figure 4 part a) and assume we have a '++' surface. Then, the surface must curve away from the viewer and continue to curve in this direction (otherwise the patch would contain a concave or hyperbolic sub-patch). This is the *bubble patch* exception. A similar argument holds if the patch has the '--' shape.

Next, consider a set of concave segments, implying a '+-' surface (see Figure 4 part b). At each vertex on the full contour, the corresponding surface must alternatively curve away and towards the viewer on opposite sides of the vertex (i.e. the vertex must be a cusp formed by the projection of an asymptotic direction), or else we would have either a crease or a tip of a 'hyperbolic cone' surface, both of which violate our C^2 assumption. Hence, we must have an even number of segments to the full contour (an odd number implies a vertex where both folding surfaces go in the same direction) and the surfaces at the corresponding full extremal boundary must alternatively curve towards and away from the viewer.

Consider now a reduced version of the patch, where the back-facing portion has been shrunk so it extends only a small amount beyond the full extremal boundary. The pro-

a) ELLIPTICAL b) HYPERBOLIC c) DEVELOPABLE

Figure 4: Possible Closed Contour Shapes

jection of this new patch boundary must lie within the full contour, as the full contour is the projection of the full extremal boundary and hence, locally, all of the projected surface must lie inside the full contour (refer to Figure 5).

Given the conclusions about alternate directions of surface bending, the new patch boundary must then lie alternatively above or below the forward-facing patch. As the patch boundary must be connected, and its projection must lie within the full contour, and that it cannot pass through the forward-facing patch, the sections of the patch boundary can only be connected up at the cusps of the full contour (as shown). This violates our assumption that the normals at the patch boundary are back-facing, as all normals along the line-of-sight through the cusp are perpendicular to the line-of-sight. Hence, a forward-facing subpatch cannot exist on a hyperbolic surface.

We lastly consider the all-straight-line case (see Figure 4 part c), which implies a developable surface. The full contour must have a corner (e.g. point Z) where two segments meet (or else there is only one segment and the patch is seen edge on and hence is neither back nor forward-facing). At this corner, the surface must become extremal in the same direction across both segments (as the patch has the same curvature signs everywhere). However, this implies that the corner is locally like the tip of a cone, or that there is a crease. Both cases violate our assumption of surface smoothness. Hence, this projection could not exist.

QED

3 Discussion

The results given above hold for orthographic projection, but extensions exist for perspective projection. Quadratic surface patches and patches taken from a torus (assuming they are from the regions with constant Gaussian curvature sign) satisfy the assumptions of the theorem. Hence, the results allow us to determine when patches of most of the types currently used for model-based vision are back-facing.

We still have the problem of determining if all boundary points are back-facing. Fortunately, this is not such a problem, as we usually have an analytic description of the boundary curve, from which we can determine the nearest surface point, and from the surface's analytic description, determine the normal. Finally, given the reference frame estimate, we can predict the orientation of that normal relative to the camera. While this still requires some effort, the computational complexity of this boundary prediction process is $O(n)$ instead of $O(n^2)$ for the surface.

We also have the problem of determining if a given model patch is a *bubble patch*. As this is independent of viewpoint, this can be determined in advance and recorded in the model.

In practice, reference frame estimates often include some uncertainity, because of measurement error and uncertainity in feature correspondence, etc. Hence, some nearly tangential surfaces (i.e. surfaces whose normals are nearly perpendicular to the line of sight) may be either totally back or forward-facing, depending on small changes in viewpoint. So, visibility analysis with real data will need to consider this problem (e.g. as in [2]).

Back-facing visibility analysis is not a complete visibility analysis, as forward-facing surfaces may still be partially or totally invisible, because of occlusion by closer object surfaces. We have detected instances of these by a ray-casting image synthesis [2], but sometimes simpler forms of on-line analysis are possible (e.g. with polyhedra). At present, to avoid the computational expense, we record the visibility information for significantly different viewpoints explicitly in a viewer-centered portion of the full geometric model [1].

When working with curved objects, there is certainly a greater possibility that a surface patch will be visible at the extrema of the object than a planar patch (that must be invisible for one-half of the viewsphere). However, except for simple objects, there will usually be enough surface patches that some will be totally back-facing, and hence the analysis presented in this paper is useful.

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